Progress in Magnetohelioseismology

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Mode conversion

In addition to Alfvén waves, magnetic atmospheres support two waves:

- **Fast MAG waves**
  - trapped acoustic waves at large depth
  - refracted at large height

- **Slow MAG waves**
  - downward travelling Alfvénic waves at large depth
  - upward travelling acoustic waves at large height

In the region where \( a \approx c \) (or \( \beta \approx 1 \)), the various waves can interact, lose their individual character, and exchange energy.
Where is the $\beta = 1$ layer?

Mathew et al (2004) computed the plasma-$\beta$ ($= p_{gas}/p_{mag} = 2c^2/(\gamma a^2)$) using two infrared Fe I lines that form 20–30km below $\tau = 1$.

- In the umbra $\tau = 1$ is depressed by 400 km (Wilson depression).
- The $\beta = 1$ layer lies below this level in umbra and some of penumbra.

$\Rightarrow \beta = 1$ layer is at least 220 km below the MDI Ni line in umbrae.
Horizontal wavenumber eigenvalues

Variation with field inclination ($B = 2$ kG, $f = 2.5$ mHz):

- Absorption ($\approx \text{Im} (k)$) by uniform inclined field peaks when $\theta \approx 30^\circ$.
- $\Delta k = k_0 - \text{Re} (k)$ ($\approx$ the phase shift) is also affected by field inclination.
Simple multi-shell model

- Apply eigenvalues $k$ to each shell (for given field strength and inclination).
- Match $u_r$ and $p_{tot}$ across each shell boundary.
- Solve for $A$, then: $\alpha = 1 - |A|^2$, $\delta = -\arg(A)$.
- We have employed a genetic algorithm to find the radial structure that produces the best fit to the Hankel analysis data (for 5-shell models).

Constraints:
- $\theta_1 = 0$ (central umbra).
- Field strength decreases (whereas the inclination increases) monotonically with radius.
- Minimum shell width: 1 Mm.
- Maximum spot radius: 30 Mm.

Ignores:
- Acoustic jacket.
- Mode-mixing.
- Variation of $B$ with depth.
- Flows.
Comparison with observations

- Best fit for $\alpha$ and $\delta$ ($\chi^2_\alpha = 5.1$, $\chi^2_\delta = 3.0$, $\chi^2_t = 4.3$):
  
  $B = \{3, 3, 3, 2, 2\}$ kG, $\theta = \{0, 45^\circ, 55^\circ, 55^\circ, 60^\circ\}$,
  
  $R = \{1.0, 4.1, 5.8, 9.2, 12.5\}$ Mm. Observations from Braun (1995).
Conclusions and consequences

- Though simplistic in some respects, our models can produce phase shifts, $\delta$, and absorption coefficients, $\alpha$, that agree well with the observations (simultaneously).
  - Corresponding field strengths $B = 1 - 3$ kG are consistent with measurements.
  - Absorption produced by mode conversion in inclined magnetic fields ($\theta \approx 30^\circ$) is typically ample to explain the observed levels (especially at higher frequencies).
    $\Rightarrow$ Though the Hankel analysis observations of $\alpha$ may be masked by high frequency emission from acoustic glories (Lindsey & Braun, 1999).

- As a function of frequency, the phase shift agreement is excellent
  $\Rightarrow$ reduced mean travel times may be magnetic in origin (no need for a sound speed or temperature enhancement below active regions).
The real parts (i.e., phase shifts) are essentially unaffected by the upper B.C.

- The imaginary parts (i.e., absorption) are sensitive to the upper B.C.
  \[ \Rightarrow \text{ramp effect: } \omega_c \rightarrow \cos \theta \omega_c. \]
Magnetic ray theory

Mode conversion causes the rays to split into fast and slow components at:
1. equipartition layer $a = c$
2. in the vicinity of the acoustic cutoff levels

$\Rightarrow$ A testable prediction from ray theory is that field inclined at $20^\circ$ should give a different atmospheric signature to field inclined at $-20^\circ$. 
Hankel analysis


Decompose oscillations in annuli around sunspots into ingoing and outgoing waves (Hankel functions).

NOAA5254

\[ r_{\text{umbra}} \approx 9 \text{ Mm}, \quad r_{\text{penumbra}} \approx 18 \text{ Mm} \]

(fairly symmetric)

NOAA5229

\[ r_{\text{umbra}} \approx 6 \text{ Mm}, \quad r_{\text{penumbra}} \approx 15 \text{ Mm} \]

(irregular)
Observed interaction

Absorption coefficient $\alpha$:

\[ \alpha = 1 - \left( \frac{|A_{\text{out}}|}{|A_{\text{in}}|} \right)^2 \]

is generally positive $\Rightarrow$ acoustic energy is lost.

Phase shift $\delta$:

\[ \delta = \arg(A_{\text{out}}) - \arg(A_{\text{in}}) \]

is positive at higher frequencies $\Rightarrow$ phase speed is increased within the spot (for modes trapped near the surface).

Horizontal wavenumber eigenvalues

Variation with frequency ($B = 2$ kG):

- Vertical field ($\theta = 0$):

- Inclined field ($\theta = 30^\circ$):
Comparison with observations

Variation with azimuthal order $m$ ($l = 288$):

- Best fit for $\alpha$ and $\delta$ ($\chi^2_\alpha = 2.2$, $\chi^2_\delta = 21.1$, $\chi^2_t = 11.6$):
  $B = \{3, 3, 2.5, 2, 2\}$ kG,
  $\theta = \{0, 10^\circ, 10^\circ, 10^\circ, 20^\circ\}$,
  $R = \{5.6, 11.5, 16.9, 21.6, 27.6\}$ Mm.

- Favours models with radii larger than observed (absorption and phase shifting regions are not the same size).
Background model

Alfvén speed, $a^2$:

Adiabatic exponent, $\Gamma_1$: