Progress in Magnetohelioseismology

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Introduction

For more than a decade it has been known that sunspots absorb and shift the phase of f- and p-modes incident upon them (e.g., Braun, 1995). Understanding the mechanism causing each of these effects is vital to the local helioseismology of sunspots (and magnetic flux concentrations in general). Recent modelling efforts by Crouch & Cally (2003, 2005), Cally, Crouch, & Braun (2003), and Crouch et al. (2005) have demonstrated that both effects can be explained by simple sunspot models provided the magnetic field inclination is taken into account.

Because the beta-equals-unity layer typically lies in the near surface layers below the sunspot photospheres, MHD mode conversion can occur. Mode conversion provides a promising absorption mechanism because the slow magnetoacoustic-gravity waves and Alfvén waves can guide energy along the magnetic field away from the acoustic cavity into the interior or, less preferentially, up into the overlying atmosphere.

In the first part of this contribution we summarise the essential findings of our most recent investigation (Crouch et al., 2005). In the last part we use a simple model for the overlying atmosphere (consisting of two isothermal slabs) to evaluate the influence of the top boundary condition and to test the validity of the boundary conditions that was employed by Crouch et al. (2005). We also show some exciting new results from Cally (2006) and Schunker et al. (2006), who have developed magnetoacoustic ray theory that may provide predictions testable by HMI.

The magnetic field

We investigate the oscillations supported by a gravitationally stratified atmosphere permeated by a straight, uniform, magnetic field of arbitrary inclination,

$$\mathbf{B} = \mathrm{B}\,(\sin\theta\,\hat{\mathbf{e}}_x + \cos\theta\,\hat{\mathbf{e}}_z)\ ,$$

where θ is the angle between the magnetic field vector and gravity (curvature of the solar surface is ignored). In solving the governing wave equations horizontal variations are neglected, but will be re-introduced later in a crude fashion.

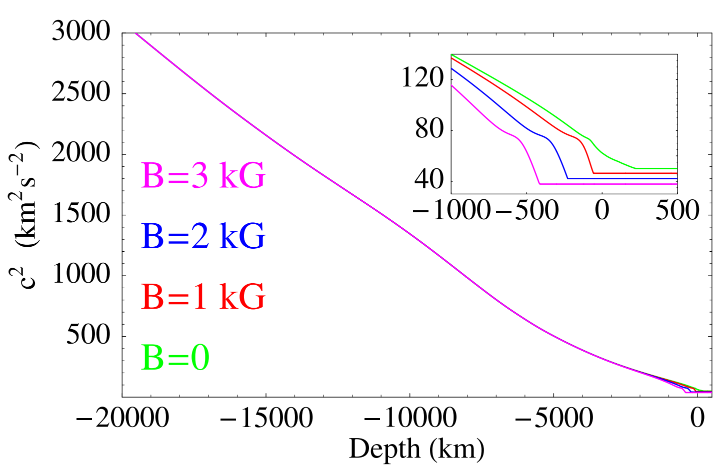
Solar interior model

Our solar interior model is based on the realistic solar model fgong.l5bi.d.15 from the Aarhus adiabatic pulsation package (Christensen-Dalsgaard, 1997), it accounts for the vertical variation of the gas pressure, density, adiabatic index, and local gravitational acceleration. We reduce the pressure in this model to account for the magnetic pressure, and to ensure total pressure balance, i.e.,

$$P_{ext} = P_{in} + \frac{\mathrm{B}^2}{2\mu}\ .$$

The modified pressure is then used to correct the adiabatic coefficient Γ_1 with a simple Saha equation solver that includes the ionisation of hydrogen and helium only.

Sound speed variation with depth



The influence of the magnetic pressure is strongest near the surface, which causes the sound speed to be reduced in models with larger field strength. On the other hand, at great depth, the gas pressure overwhelms the magnetic pressure and so there is little difference between the sound speed profile of the various models. Details of the variation in Alfvén speed and adiabatic exponent can be found in Crouch et al. (2005).

Upper boundary conditions

At this stage, we will not concern ourselves with modelling wave propagation in the upper atmosphere (which is complicated by field non-uniformity and non-linear wave dynamics Rosenthal et al., 2002; Bogdan et al., 2003). Consequently, at the top of each model ($z = 500$ km), we impose rigid lid boundary conditions (each component of the displacement vector vanishes).

Lower boundary conditions ($z \to -\infty$)

At a depth of $z = -16$ Mm we append a shifted adiabatic polytrope (with $m = 3/2$, or equivalently $\gamma = 5/3$). This allows asymptotic solutions for the wave equations can be developed (at $z \to -\infty$), which allow us to impose physical boundary conditions: wave-like disturbances must be outgoing and evanescent modes must decay (see Crouch & Cally, 2003, 2005, for details).

In the quiet Sun model this selects the trapped acoustic wave.

For the magnetic cases, the lower boundary conditions select:

- the downward travelling slow MAG wave, which carries energy away from the conversion layer and is responsible for any absorption displayed by our model; and
- the (acoustic) trapped fast MAG wave, which is very similar to the non-magnetic p-mode but modified by the magnetic field.

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Active region 10030 image courtesy of Royal Swedish Academy of Sciences

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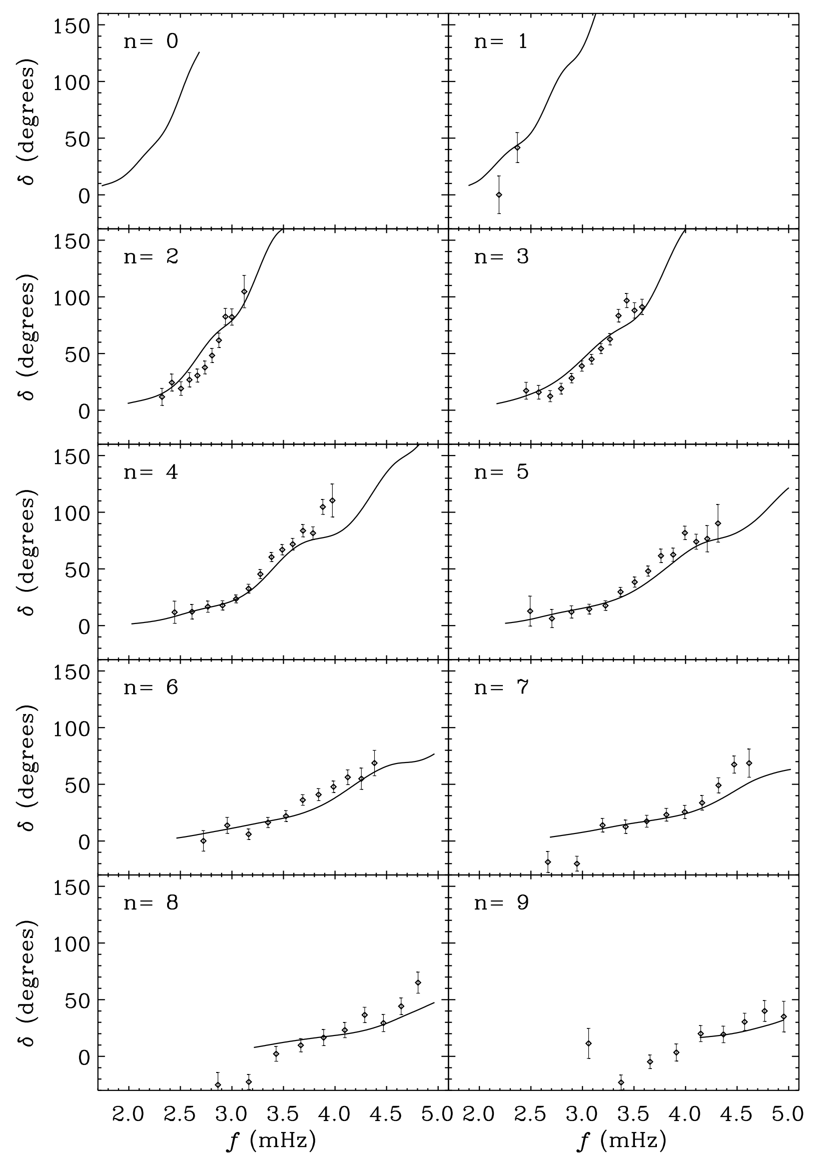
Genetic magnetohelioseismology

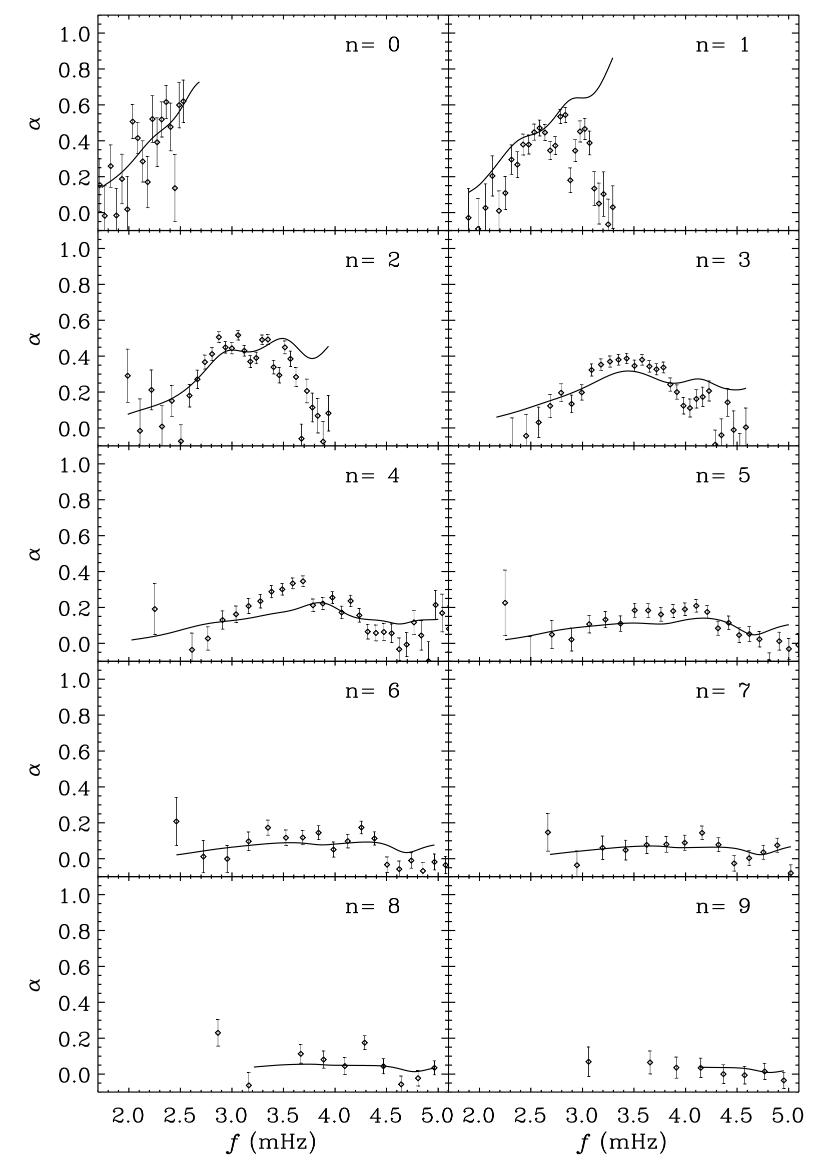
To account for the observed radial variation of the magnetic field strength and inclination across a sunspot, we construct a simple model of concentric cylinders (or shells). Given the field strength, field inclination, and radial thickness of each shell, the eigenvalues can be used to calculate the resultant absorption and phase speed change produced by the model. We use a genetic algorithm to iteratively adjust the model parameters controlling the radial structure in order to find the best fit with observations.

Comparison with observations – phase shifts



Phase shift δ as a function of frequency for axisymmetric modes ($m = 0$) with radial orders $n = 0, \ldots, 9$. The *full curves* are the theoretical predictions produced by a best fit 5-shell model with: B= $\{3, 3, 3, 2, 2\}$ kG, $\theta = \{0, 45°, 55°, 55°, 60°\}$, and R= $\{1, 4.1, 5.78, 9.23, 12.53\}$ Mm. The *diamonds* (and associated error bars) are the observed m-averaged phase shifts for NOAA5254 (Braun, 1995). Only observational data points which satisfy $\sigma_\delta < 18°$ are displayed.

Comparison with observations – absorption coefficients



Same as the plot above, except the absorption coefficient α is plotted, and only observational data points which satisfy $\sigma_\alpha < 0.2$ are shown. Both plots are from Crouch et al. (2005).

Consequences for sunspot seismology

It is a commonly held perception that the observed phase speed increase is indicative of an enhanced sound speed in the subphotospheric layers of sunspots (see also Fan et al., 1995; Duvall et al., 1996; Kosovichev et al., 2000). Our results show that this is not the only explanation – those effects may be primarily magnetic in origin. The sound speed in our magnetic models is actually slightly reduced in comparison to the non-magnetic quiet Sun model. Yet the resultant phase shifts produced by our simple sunspot models can agree remarkably well with the observations (depending on the specifics of the radial structure). In addition, the absorption from our models is generally ample to account for the observations. Further work is needed to determine if these conclusion holds true in more complex models.

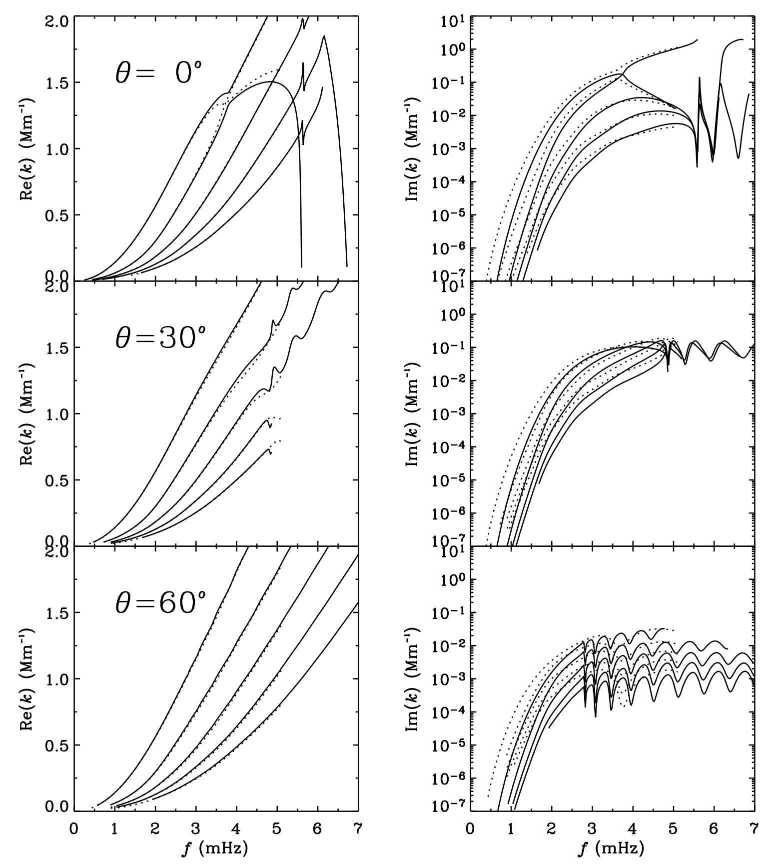
Testing the upper boundary condition

In this section we present some preliminary results from a model that relaxes the rigid lid boundary condition (as employed by Crouch et al., 2005). Despite their limitations in the atmosphere, as a first approximation we retain the assumptions of field uniformity and small amplitude waves. We model the atmosphere above the photosphere with two isothermal slabs. The gravitational acceleration is constant throughout the atmosphere and matches the value at the top of the solar interior model ($z = 500$ km). The lowest slab (which is intended to mimic the chromosphere) extends from $z = 500$ km to $z = 2000$ km, and has a temperature and adiabatic exponent that match the values at the top of underlying interior model. The upper slab is semi-infinite and has a temperature $T = 2 \times 10^6$ K (consistent with values observed in corona) and adiabatic exponent $\gamma = 5/3$.

Solving the wave equations in the atmosphere

It turns out that the linear wave equations for an isothermal slab permeated by a straight, uniform, inclined magnetic field can be solved analytically, when the direction of wave propagation is parallel to the vertical plane containing the magnetic field (i.e., $\phi = 0$). In a similar fashion to the vertical field case (that was studied by Cally, 2001), the solutions for inclined field problem can be written in terms of the hypergeometric ${}_2F_3$ functions (it should be pointed out that Zhugzhda & Dzhalilov, 1984, solved the same equations in terms of Meijer functions but these are significantly more complicated than the ${}_2F_3$ functions). We impose physically consistent boundary conditions at $z \to \infty$, such that wave-like disturbances travel upward and evanescent modes decrease with increasing height. The fast MAG waves are refracted at great height by the increasing sound speed and have leading behaviour $\exp(-kz)$. The slow MAG waves have leading behaviour $\exp\left\{-(z/2H)\left[1 + 2ikH\cos\phi\tan\theta - \sec\theta\sqrt{\cos^2\theta - (\omega/\omega_{ac})^2}\right]\right\}$, which shows that the acoustic cutoff frequency ω_{ac} is effectively reduced by a factor of $\cos\theta$ in regions with inclined magnetic field (in comparison to the quiet Sun value De Pontieu et al., 2004).

Preliminary eigenvalue results



Summary – phase shifts

One particularly important point demonstrated by the plot above is that the real parts of the wavenumber eigenvalues are fairly insensitive to the specifics of the upper boundary condition (in most cases the curves of the two models are indistinguishable). The real parts of the wavenumbers control the phase shift produced by the simple shell models. Consequently, our conclusion regarding the interpretation of the phase speed changes is unmodified (though the influence of field non-uniformity and non-linear wave dynamics are yet to be tested).

Summary – absorption

On the other hand, the imaginary part of the wavenumbers (which control the absorption) are more sensitive to the nature of the upper boundary condition. At lower frequencies it appears that the magnitude of $\mathrm{Im}(k)$ is actually slightly reduced in the model that allows waves to propagate upward. At higher frequencies the difference between the two models is quite dramatic. The oscillations of $\mathrm{Im}(k)$, exhibited by the model with an atmosphere, are due to standing wave resonances in the cavity between the temperature minimum region ($z \approx 500$ km) and the transition region (the discontinuity separating the chromosphere and corona at $z = 2000$ km). It is evident that the resonances occur at lower frequencies in models with larger field inclination. This is to be expected because the acoustic cutoff frequency f_{ac} is inclination dependent (as explained above). For the $B = 2$ kG model, $f_{ac} = 5.46$ mHz in vertical field, $f_{ac} = 4.73$ mHz when $\theta = 30°$, and $f_{ac} = 2.73$ mHz when $\theta = 60°$. Those values are in very good agreement with the frequencies where the resonances commence. Waves below these frequencies will not form resonances as they do not propagate in the model chromosphere. Apart from the resonances, the average overall magnitude of the imaginary parts is quite similar to the rigid lid model (though the rigid lid values tend to provide an upper bound).

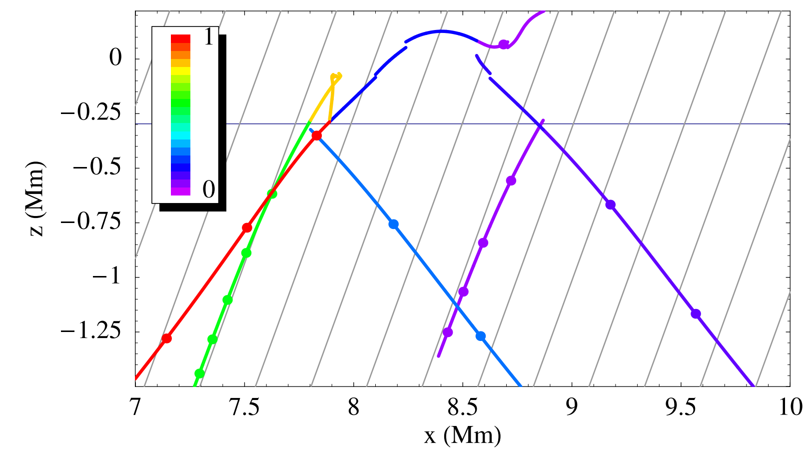
Ray theory

Magnetoacoustic ray theory identifies two kinds of mode conversion points, where rays split into fast and slow magnetoacoustic waves:

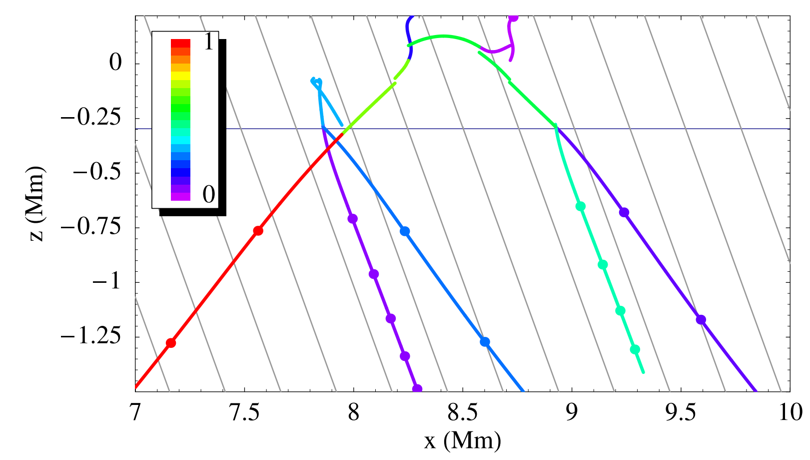
- Type 1, near the equipartition surface where the sound and Alfvén speeds coincide, and
- Type 2, in the vicinity of the acoustic cutoff levels.

Schunker et al. (2006) find that mode conversion is important in sunspots wherever the magnetic field is sufficiently strong that the Alfvén/acoustic equipartition level lies below the steep rise in acoustic cutoff frequency at roughly -100 km. If this holds, it is found that waves approaching the surface at a large angle to the magnetic field easily penetrate the acoustic cutoff barrier to enter the atmosphere as travelling waves. On the other hand, waves meeting the field lines at fine angles are reflected by the cutoff barrier, and will therefore be evanescent above.

Ray paths for field inclination $\theta = 20°$



Ray paths for field inclination $\theta = -20°$



Summary

- Ray theory confirms the findings of eigenvalue calculations in that field inclination is a vital ingredient in modelling active region acoustics.
- A testable prediction from ray theory is that field inclined at $20°$ should give a different atmospheric signature to field inclined at $-20°$.

Future directions

Models of this type, that account for the magnetic fields in active regions in a simple yet reasonable manner, will be critical for interpretation of data expected from the upcoming Helioseismic and Magnetic Imager (HMI) onboard the NASA Solar Dynamic Observatory mission.

Indeed, modelling based on the mode conversion hypothesis may yield testable predictions that could be examined by HMI.

There are several questions that we intend to investigate:

1. How much acoustic energy is channelled from the p-modes into the solar atmosphere? Is it enough to contribute to chromospheric or coronal heating?
2. What are the resultant absorption coefficients (especially at higher frequencies) and how do they compare with observations?
3. Can useful information about the mode conversion process be extracted by comparing the displacement vectors (mode amplitudes and phase perturbations) in the photospheric layers with observations (at $z \approx 200$ km), and how sensitive is this to the details of the atmosphere model?
4. Can these models be used to interpret observations in penumbra (highly inclined field models)?

In addition, the information gained from these simple models will guide the development, testing, and interpretation of results from 2-D and 3-D numerical MHD models which incorporate more sophisticated magnetic geometries, thermal perturbations, and flows.

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