Determination of the coronal density stratification from harmonic coronal loop oscillations

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Introduction

- Observations: Multi mode kink-oscillations of coronal loops (Verwichte et al. 2004).
- Theory: Computations of eigenmodes of longitudinally stratified loops (Andries et al. 2005).

$\Longrightarrow$ **New seismological tool!**

Observations

Verwichte et al. (2004) have detected the simultaneous presence of both the fundamental and the first overtone kink-mode in two loops. The ratio of the periods was found to be: 1.81 and 1.64 respectively. They seem to be worried about the numbers differing from 2 and from an analysis of the errors they conclude that the observed values are not in conflict with the expected value of 2.

Theory

Higher harmonics need not necessarily oscillate with a multiple of the fundamental frequency. Guitar players know very well that strings can be bad because e.g. the mass distribution along the string is not perfectly homogeneous. Bad strings make it impossible to tune the guitar well. When pressing e.g. the fret at halve of the string length, the frequency should be exactly double that of the fundamental. If not, all you can do is tune the guitar so that the fundamentals sound right but then the high tones sound false. As the loops in question stretch out into the solar atmosphere up to about one density scale height, it should not come as a surprise that they seem to be tuned badly.

Andries et al. (2005) have recently calculated fundamental kink-mode periods in longitudinally stratified loops and have shown that to a good approximation the frequency is unaffected by longitudinal density stratification itself but only depends on the weighted mean density. The weight function is not surprisingly given by the square of the fundamental sine, as this represents the longitudinal wave energy distribution along the loop. Likewise, it could be anticipated that the second harmonic is only influenced by the mean density when weighted by the square of the second sine. Clearly, different harmonics are influenced differently by the density distribution along the loop, and by inversion, observation of the frequencies of different harmonics can give us information about the possible density distribution along the loop.

Model

Equilibrium:

- no fluid pressure ($\beta = 0$), straight and constant magnetic field
- a cylindrically symmetrical (tube) density enhancement $\rho(r, z)$ with an internal $0 < r < a$ and an external $r > b$ region where $\frac{\partial \rho}{\partial r} = 0$
- a longitudinally stratified density profile $\rho(r, z)$

Linear perturbations:

- $\sim \exp(\imath m\varphi)$ $\quad m = 1$ kink mode oscillations
- $\sim \exp(-\imath\omega t)$ $\quad \omega = \frac{2\pi}{\text{Period}}$

Longitudinal stratification

We assume an exponentially stratified atmosphere:

$$\rho(h) = \rho_0 \exp(-h/H)$$

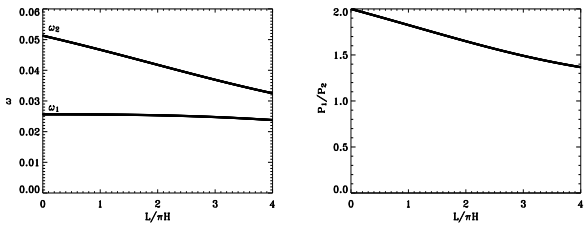
h : height in the solar atmosphere
H : density scale height
Projected on a semi-circular loop of length L with a height of consequently L/π:

$$\rho(z) = \rho_0 \exp(-L \sin(\pi z/L)/\pi H)$$

$L/\pi H$: measure of the stratification $\Rightarrow$ independent variable on the figures.

Fundamental and first overtone frequencies

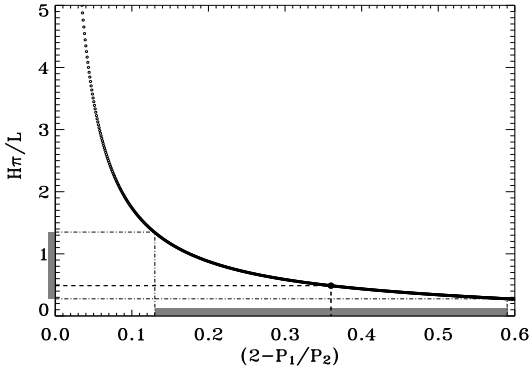
Frequencies obtained with the method described by Andries et al. (2005)



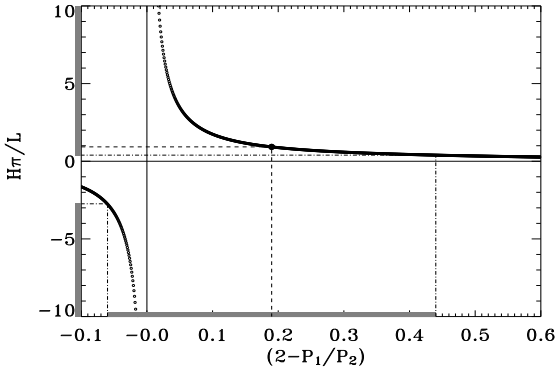
(a) The frequency of the fundamental kink mode and its first harmonic of a tube with length $100R$ and density contrast 2, as a function of the density stratification $L/\pi H$. The frequencies are normalized with respect to the mean density weighted with $\sin^2 \frac{\pi}{L} z$. (b) The ratio of the periods P_1/P_2 or frequencies ω_2/ω_1 of the fundamental kink mode and its first harmonic of the same tube.

Coronal loop seismology

By inversion the measurement corresponding to path D in Verwichte et al. (2004) yields:
$H \approx 36Mm \in [20Mm; 99Mm]$ *Reasonably confined estimate*



The measurement corresponding to path C in Verwichte et al. (2004) yields:
$H \approx 65Mm > 27Mm$ *Stratification cannot be extremely strong*
or
$H < -190Mm$ *Higher density at the loop tops possible but not by more than 44%*



Summary and Conclusions

- The period ratio of the fundamental kink mode of a coronal loop and of its first harmonic differs from 2 if there is longitudinal density stratification along the loop.
- The observed ratio can be used to provide estimates of the density scale height in the solar atmosphere.
- The two known observations yield ratios that are lower then 2 but only in the first case the deviation from 2 is significant. Hence, the second case cannot exclude the possibility of heavier plasma at the loop tops, but it does limit that possibility considerably. Both observations are consistent with an expected scale height of about 50Mm. For the first case we obtain a reasonably confined estimate of the density scale height between 20Mm and 99Mm.
- More and more accurate simultaneous observations of the fundamental kink mode and its harmonic in coronal loops are necessary, before this method can be used to provide accurate quantitative results.
- We thus want to emphasize that the observation of different harmonics in the same coronal loop provides an important new tool for the determination of the longitudinal density profiles specifically, and for coronal seismology in general.