LightMachinery

HMI Beam Splitter Phase Calculations (II)

Introduction

This note is an update on my note of November 20. I have thought about the phase in reflection and transmission in more detail, and based on the design of the first beam splitter coating, I have made some calculations of the effect.

The first main concept is that the phase for a beam reflected from or transmitted through a thin film coating varies as a function of incident angle. In general, the effect of incident angle is significantly stronger for the transmitted p- light than for reflected s- light, and as the coating is made thicker, the difference increases.

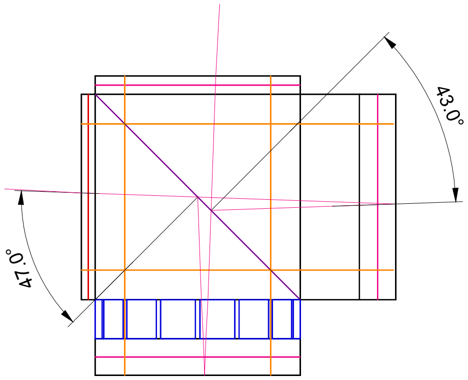
The second main concept is that the Michelsons are part of an imaging instrument, and each pixel is the superposition of a series of rays that pass through the Michelsons with a range of different angles. The angle of the central ray for each pixel also varies over the field of view of the instrument.

In combination, these two effects will cause the observed I0 to vary in the sagittal plane, and will cause a reduction in the observed brightness. I don’t know if the reduced brightness will be observed directly, or if it will appear as reduced contrast. The measured performance of the MDI Michelsons should be compared to the predictions that I make in this note.

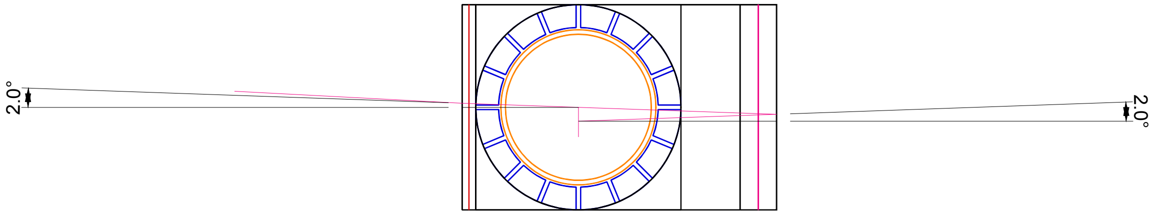
The rest of the note explains the origin of these effects, and estimates their magnitudes.

Beam splitter phase

The first point to note is that sagittal rays and tangential rays behave differently (see the sketch below).



Sagittal rays



Tangential rays

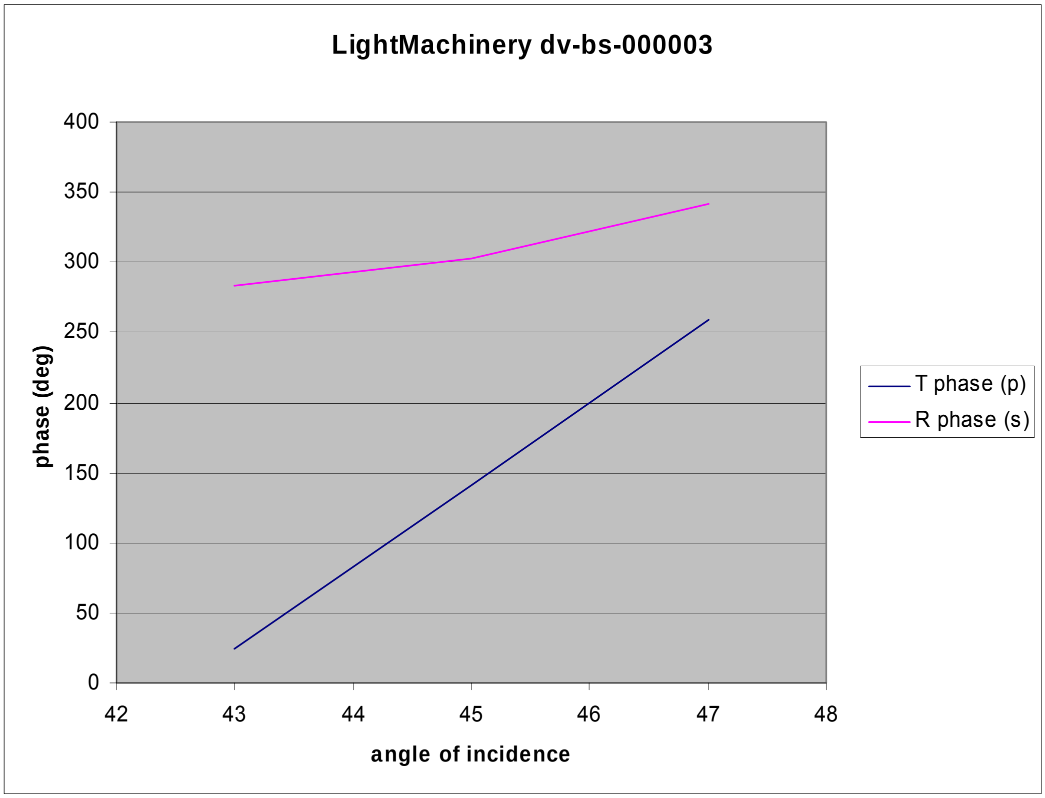
The incident angle of sagittal rays is not the same for the initial reflection from the beam splitter, and for the subsequent transmission. For example, if the incident angle is 47° for the initial reflected ray, then the transmitted ray has an incident angle of 43°. However, a tilt in the tangential plane does not change on reflection.

The contribution from the beam splitter coating to the phase of the two beams in the interferometer is not symmetric. For the rays in the sagittal plane passing through the vacuum arm:

$\varphi_v = \varphi_{R(45+\theta)} + \varphi_{T(45-\theta)}$, but for the rays passing through the solid arm:

$\varphi_s = \varphi_{T(45+\theta)} + \varphi_{R(45-\theta)}$.

The next chart shows the predicted variation in phase in the first beam splitter test coating. Clearly the slope of the transmission phase with angle is much steeper than for the reflection phase. This effect can be visualized by thinking about how light interacts with the coating. In transmission, light passes all the way through the coating, but in reflection, the light appears to be reflected from some intermediate plane (less than half way through the coating).



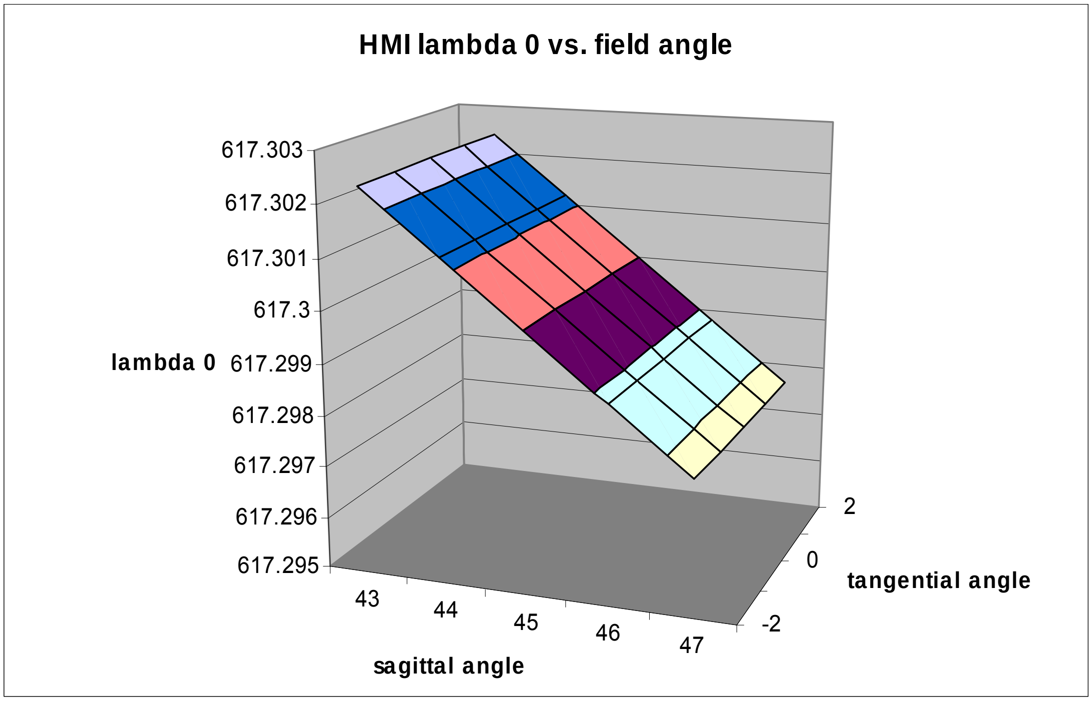
For rays in the tangential plane, there is a similar relation, but because of the symmetry, there is no variable phase shift between the vacuum leg and the solid leg:

$\varphi_v = \varphi_{R(\psi)} + \varphi_{T(\psi)}$, and for the rays passing through the solid arm:

$\varphi_s = \varphi_{T(\psi)} + \varphi_{R(\psi)}$.

The net effect on the interferometer is a uniform phase gradient across the field of view in the sagittal plane. As an imaging instrument the observed effect will be less because the rays that interfere at each camera pixel have a range of angles corresponding to the f/number of the system. The phase variation will be detected as a variation in λ_0 as a function of the sagittal angle.

This coating was designed to have maximum discrimination between polarization states, and we did not constrain the phase variations. As a result, the phase variation from this coating is quite extreme. For this coating, λ_0 will decrease by about one quarter of the FSR between the center and the extreme edge of the image. The next chart shows the predicted effect for the narrow band Michelson.



The effect for the wide band Michelson will be the same as a proportion of FSR, but the absolute effect will be twice as large.

There is a secondary problem that results from the phase difference between the rays that combine to form each pixel. The intensity will be reduced as a function of the phase range. For the sample coating with a predicted phase variation of about 180°, the intensity is reduced to about 60%. Mathematically, this is the sum of a series of sine waves with a smoothly varying phase shift, and the larger the total phase shift, the smaller the resulting intensity. The contrast will be reduced somewhat, and the overall brightness will be poorer than expected.

I have done similar calculations for the MDI Michelsons, based on a rough estimate of the coatings that were used for their beam splitters. These coatings have fewer layers than the test coating for the HMI beam splitter, so the predicted effect is less. I expect the gradient in λ_0 across the full sagittal field (±2°) would be about one quarter of the FSR for the MDI Michelsons. How does this match up with the measured numbers? The intensity for the MDI Michelsons is predicted to be about 85%.

There is another minor effect for increasing tangential ray angles. The beam will be slightly depolarized (0.5% for the worst case double passed), because the plane defined by the incident and reflected beams rotates slightly relative to the cube axes, leading to mixing. This sets an effective limit on the amplitude properties of the polarizing film.

Coating Optimization

I think we should re-optimize the coating to minimize the phase variation of the coating to see what the trade offs are between the amplitude and polarization effects. We will look at a range of different coating designs to determine the practical trade off between amplitude and phase performance and coating yield. Section 3.6 of the Technical Specification requires a variation of λ_0 between the center of the field and the extreme of 5.8% of the FSR or less, and this specification may not be achievable because of the coating phase effect. The table below shows theoretical results for beam splitter coatings with different layer counts. An approximation of the MDI coating, and the test coating are shown for reference. The figure of merit (FOM) I used the product of $(1\text{-}Tp)^2$ and the image brightness. The phase spread translates to a variation in λ_0 across the field of view, and based on the MDI experience can probably be calibrated out. I don't know what the trade offs are between total brightness and λ_0 shifts.

Coating performance vs. layer count

layers	max Tp	Min Rs	phase spread (degrees)	image brightness due to phase spread	FOM
7	10%	92%	36	97%	7857
9	5%	96%	47	96%	8664
11	1.30%	99.10%	64	94%	9157.189
15	0.70%	99.40%	76	92%	9071.651
17	0.20%	99.70%	85	90%	8964.036
MDI (21)*	0.10%	99.90%	100	85%	8483.009
Test coating (about 40)	0.10%	99.90%	180	60%	5988.006

Phase contribution from cement layer

There is another smaller phase effect in the beam splitter which arises from the difference in refractive index between the cement and the glass. This effect is roughly parabolic as a function of sagittal angle with the phase maximum occurring at 45°. Assuming a cement layer thickness of 15 µm, the phase difference between the center and extreme rays is approximately 0.035 waves. In terms of λ_0 shift this is equivalent to 1.8% of the FSR. This effect scales with the thickness of the cement layer, and clearly the cement layer thickness should be minimized. The magnitude of the phase effect is not affected by the location of the cement layer (either before or after the coating), but the sign of the phase shift will depend on the cement layer position.