HELIPOSEISMIC MODELING FOR DETERMINATION OF LARGE-SCALE AND MERIDIONAL FLOWS

Thomas Hartlep

Collaborators:

Junwei Zhao¹, Nagi N. Mansour², Alexander G. Kosovichev¹, Tom L. Duvall³, Markus Roth⁴, Hans-Peter Doerr⁴

¹Stanford University, ²NASA/Ames, ³NASA/Goddard, ⁴Kiepenheuer-Institut für Sonnenphysik
(A) NUMERICS

- Numerical simulations of acoustic wave propagation in a 3D model of the full sun (Hartlep et al. 2008)
- The present simulations accounts for sound speed variations, solar rotation, and the effects of mass flows
- The simulations solve linearized equations for waves propagating through a stationary background medium:

\[
\begin{align*}
\partial_t \rho' &= -\nabla \cdot \vec{m} + S \\
\partial_t \vec{m} &= -\nabla c_0^2 \rho' \rho' \vec{g} + \rho' (\vec{v}_0 \cdot \nabla \vec{v}_0 + \vec{v}_0 \nabla \cdot \vec{v}_0) + \vec{v}_0 (\vec{v}_0 \cdot \nabla \rho') \\
&\quad - (\vec{v}_0 \cdot \nabla \vec{m} + \vec{m} \cdot \nabla \vec{v}_0 + \vec{m} \nabla \cdot \vec{v}_0 + \vec{v}_0 \nabla \cdot \vec{m})
\end{align*}
\]

with \( c_0, \rho_0, v_0, g_0, \) and \( \rho' \) denoting background sound speed, density, flow velocity, and gravity, and the fluctuations of the density associated with the waves, respectively. And, where \( m \) are the momentum variations:

\[
\vec{m} = \rho' \vec{v}_0 + \rho_0 \vec{v}'
\]

- A forcing term \( S \) mimicking the excitation of acoustic waves has been added since the equations are linearized
- Equations are solved numerically using a pseudo-spectral method with spherical harmonic functions for the angular dependencies and B-splines for discretizing the radial direction
- Present simulations contain spherical harmonic degrees from \( l=0 \) to \( l=170 \)
- Background state uses the sound speed and density profiles from standard solar model \( S \) of Christensen-Darlsgaard et al. (1996) matched to a chromosphere model by Vernazza et al. (1981)
- Localized variations in the sound speed as well as mass flows can be prescribed
- Oscillations are assumed to be adiabatic, and are driven by randomly forced density perturbations near the solar surface
- Non-reflecting boundary conditions are applied at the upper boundary of the simulation domain by means of an absorbing buffer layer with a damping coefficient that is zero in the interior and increases smoothly into the buffer layer
- Perturbations of the gravitational potential are neglected, and the adiabatic approximation has been used
- To make the linearized equations convectively stable, we neglect the entropy gradient of the background model. The calculations show that this assumption does not significantly change the propagation properties of acoustic waves including their frequencies, except for the acoustic cutoff frequency, which is slightly reduced.
**B) MERIDIONAL FLOW MODELS**

- True strength and shape of solar meridional flow is unknown
- These simulations are used to learn how to measure such small and deep flows in the Sun

**Model A:**

- A meridional flow consisting of a single cell in each hemisphere
- Maximum velocity near the surface of about 20 m/s
- Reversal at a rather shallow depth of approximately 50 Mm
- Solar rotation including a realistic differential rotation model are taken into account in this simulation
- 51-hour long time-series

- Various models of meridional flows have been proposed, and we plans eventually perform simulations for a few different models to provide a basis for testing helioseismic measurements
- Here we show preliminary results for two models
(B) MERIDIONAL FLOW MODELS (CONT.)

- True strength and shape of solar meridional flow is unknown
- These simulations are used to learn how to measure such small and deep flows in the Sun

Model B:

- Measuring such small flows is very challenging, in particular using fairly short time series as can be generated by numerical simulations
- Tom suggested to improve signal-to-noise by artificially scaling up the flow velocity
- So, we took a model from Rempel (2006) and scaled up the meridional flow velocity to 500 m/s
- Solar rotation was not taken into account in this simulation
- Time-series from this simulation is now 34 hours long

- Various models of meridional flows have been proposed, and we plan to eventually perform simulations for a few different models to provide a basis for testing helioseismic measurements
- Here we show preliminary results for two models
Cross-Correlation and Fitting

- Compute cross-correlation between two points at the same longitude, an equal distance north and south of a central latitude

- Average the cross-correlation over all longitudes

- To further improve the signal-to-noise ratio, combine cross-correlations with different central latitudes within ±15 degrees

- No phase-speed filters are used

- Northward and southward cross-correlations are then fitted separately using the Gabor wavelet

- Phase travel time difference between the two signals is computed

---

**Fig.**: Time-distance diagram of the cross-correlation between the radial oscillation velocities north and south of a central latitude of +58° (averaged over ±15° latitude and all longitudes).
Preliminary Results

- Results are quite noisy
- Fit errors are lowest and the variations in travel time difference from one travel distance to another are smallest for distances around 9 degrees
- Much shorter distances are a problem because of the relatively low resolution of the simulation which does not contain very high spherical harmonic degrees
- Much longer distances are also much harder to measure because the effect of the meridional flow on the acoustic travel time of these deep traveling waves is very small

Fig.: Preliminary travel-time differences between northward and southward traveling waves as a function of latitude from the simulation of model A. Each value is an average over all longitudes, over approx. 30° in latitude, and over travel distances between 6.3° and 9.8°. The standard error (standard deviation divided by the square root of the number of samples) computed from the measurements for different travel distances is used for the error bars. Other errors have not been taking into account, so the total error is actually larger. For comparison, the dotted curve shows the expected travel-time differences for a travel distance 8° computed from the model flow using the ray path approximation. The dashed curve is its 30° running average.
Cross-Correlation and Fitting

- Here, procedure similar HMI time-distance pipeline
  - Data is remapped using Postel’s projection
  - Cross-correlations are computed in annuli with radius ranges 3°-6°, 6°-9°, 9°-12°, and 12°-15° heliographic degrees
  - Cross-correlations for the same central latitude are combined since we are interested in the rotationally averaged meridional flow
  - Cross-correlations are fitted using a Gabor wavelet to get acoustic travel times
Preliminary Results

Fig.: Preliminary travel-time measurements for simulation model B as a function of latitude and for 4 different ranges of annulus radii. For comparison, dotted curves show the travel-time differences expected from integrating the flow model along the ray paths.
Using a Fourier-Legendre spectral decomposition technique, they performed inversion of the data from model A.

Fig.: Inverted meridional flow using 360 x 90 degree patches at latitudes +45° (left) and -45° (right). Red lines indicate the true prescribed meridional flow in the simulation of model A (positive means northward). The horizontal error bars give the FWHM of the averaging kernels, the vertical error bars are due to error magnification of the frequency measurements.
**Fig.:** Inverted meridional flow for 360 x 30 degree patches (left) and 360 x 16 degree patches (right). The greater patch geometry allows probing to depths of about 20 Mm, the smaller for depths down to about 15 Mm. The arrows indicate a poleward flow on each hemisphere. Inversion results are positioned such that the averaging kernels do not overlap. For the larger patches the results seem to be more stable than for the smaller patches. In both cases the minimum depth that can be probed is approx. 8 Mm due to the missing high-degree modes in the simulation.
Simulation data is openly available and can be downloaded from my website:

http://sun.stanford.edu/~thartlep/Artificial_Data.html

I’m also working on putting data into the HMI/AIA JSOC catalog. I created a test series su_thartlep.simulation_PROD_13MAY11_MC_Rempel_Boost. Somebody should try it out to see if it has all necessary keywords they need.

REFERENCES