CR 2100: meridional flow measurements with Fourier-Legendre decomposition technique and ring-diagram analysis

H-P Doerr¹, K. Glogowski¹, A. Zaatri²,¹ and M. Roth¹

¹ Kiepenheuer-Institut für Sonnenphysik, Freiburg, Germany
² Centre de recherche en astronomie, astrophysique et géophysique, Algiers, Algeria

LoHCo Meeting August 17–19 2011
Stanford University
The Fourier-Legendre decomposition (FLD) technique

- introduced by Braun et al. (1987) as a tool to measure $p$-mode absorption in sunspots
- several applications to meridional flow measurements later on

**Advantage**
- adopted to spherical symmetry of the sun
- sensitive to low-$l$ modes
- can be applied to 'semi-global' helioseismic data: average deep flow on one hemisphere...
- ...as well as for local data: latitudinal dependence of near-surface flow
FLD applied for meridional flow measurements: basic idea

Expand surface oscillation signal in a series of equatorward and poleward travelling wave fields.

Frequencies of both components can be extracted from doppler imaging data (similar to SHT, but with modified basis functions).

Frequency shift between both components related to meridional flow by (Gough & Toomre, 1983):

\[
\Delta \nu_{nl} = C \int_{0}^{R_{\odot}} \langle v(r) \rangle K_{nl}(r) \, dr.
\]  

(1)

Mean horizontal component of meridional flow at a given depth, \( \langle v(r) \rangle \), can be inverted from (1) using appropriate sensitivity functions \( K_{n,l}(r) \).
Data set

Data: 1024 x 1024 binned HMI doppler images of CR 2100, same effective patch size for FLD and rings. But for FLD we use stripes rather than square 16x16 degree tiles as for the rings.
FLD Procedure: frequency determination

Power spectra are computed for all timeseries $A_{l,m}(t)$ and $B_{l,m}(t)$ of the poleward, equatorward travelling wave fields.

Time series are split in chunks of approx. 7d length. For fitting, we use the average of all chunks and also the $m$-averaged spectra. Measure frequencies with M-L fitter, leakage is currently not considered.
The frequency shifts are then inverted for the meridional flow with a SOLA method (Pijpers & Thompson, 1992).

Kernels are comparably well-shaped and located closely to their target depths.
FLD flow velocities

Meridional Flow during CR 2100 using FLD

- 2.0
- 3.1
- 4.4
- 5.8
- 7.1
- 8.5
- 10.2
- 11.6
- 13.1
- 14.3
- 15.8

Latitude [deg]

M. Roth

CR2100: FLD meridional flow measurements

LoHCo Meeting August 17–19 2011
Meridional Flow during CR 2100 using Ring Diagrams
Results

FLD vs. Rings @ 2 Mm

Meridional Flow during CR 2100, depth: 0.9971 (2.0 Mm)

Ring Diagram
Fourier Legendre

M. Roth

CR2100: FLD meridional flow measurements

LoHCo Meeting August 17–19 2011
Results

FLD vs. Rings @ 7 Mm

Meridional Flow during CR 2100, depth: 0.9898 (7.1 Mm)

Ring Diagram
Fourier Legendre

M. Roth

CR2100: FLD meridional flow measurements

LoHCo Meeting August 17–19 2011
Results

FLD vs. Rings @ 12 Mm

Meridional Flow during CR 2100, depth: 0.9833 (11.6 Mm)

Ring Diagram
Fourier Legendre

M. Roth
CR2100: FLD meridional flow measurements
LoHCo Meeting August 17–19 2011
FLD vs. Rings @ 16 Mm

Meridional Flow during CR 2100, depth: 0.9773 (15.8 Mm)

Legend:
- Ring Diagram
- Fourier Legendre
Inversions for the deep flow with artificial data

We applied FLD technique to one of the artificial helioseismic datasets from T. Hartlep (full-sun including simple models of flows, $T \approx 2d$ solar time).

To demonstrate potential of the FLD technique, we use the full hemispheres (patch size of 360x90 deg), centered on $\pm 45$ deg.

vertical errorbars: FWHM of SOLA kernels
horizontal: error propagation from frequency fits
Conclusions

- FLD flow velocities are comparable to those from ring-diagram
- FLD is sensitive throughout solar convection zone
- Tests with artificial data show great potential for measurements of the deep flow
- However, systematic effects need to be studied in detail
- Need for better calibration of inversion for deep flow measurements
- ...waiting for longer time series of artificial data
Normal Mode Decomposition based on Legendre Polynomials

Expand oscillation signal as superposition of travelling waves propagating in opposite directions (in-/outward)

\[ \Psi(\theta, \phi, t) = \sum_{l,m,\nu} [A_{l,m,\nu} X^m_l(\theta) + B_{l,m,\nu} X^{m*}_l(\theta)] e^{i(m\phi + \nu t)} \]

Legendre functions as basis functions:

\[ X^m_l(\theta) = N_{l,m} \left[ P^m_l(\cos \theta) - \frac{2}{i\pi} Q^m_l(\cos \theta) \right] \]

Extract \( A \) and \( B \) from measured

\[ A_{l,m}(t) = C_{\phi_{\text{max}}} \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \int_{\theta_{\text{max}}}^{\theta_{\text{min}}} \Psi(\theta, \phi, t) X^m_l(\theta) e^{-im\phi} \sin \theta \, d\theta \, d\phi \]

Power-spectra of \( A_{l,m}(t) \) and \( B_{l,m}(t) \) show frequency shift in presence of flows
Normal Mode Decomposition based on Legendre Polynomials

Expand oscillation signal as superposition of travelling waves propagating in opposite directions (in-/outward)

\[ \Psi(\theta, \phi, t) = \sum_{l,m,\nu} \left[ A_{l,m,\nu} X_l^m(\theta) + B_{l,m,\nu} X_l^{m*}(\theta) \right] e^{i(m\phi+\nu t)} \]

Legendre functions as basis functions:

\[ X_l^m(\theta) = N_l^m \left[ P_l^m(\cos \theta) - \frac{2i}{\pi} Q_l^m(\cos \theta) \right]. \]
Normal Mode Decomposition based on Legendre Polynomials

Expand oscillation signal as superposition of travelling waves propagating in opposite directions (in-/outward)

$$\Psi(\theta, \phi, t) = \sum_{l,m,\nu} \left[ A_{l,m,\nu} X_l^m(\theta) + B_{l,m,\nu} X_l^{m*}(\theta) \right] e^{i(m\phi+\nu t)}$$

Legendre functions as basis functions:

$$X_l^m(\theta) = N_l^m \left[ P_l^m(\cos \theta) - \frac{2i}{\pi} Q_l^m(\cos \theta) \right].$$

Extract $A$ and $B$ from measured $\Psi$

$$A_{l,m}(t) = C \int_{\phi_{\min}}^{\phi_{\max}} \int_{\theta_{\min}}^{\theta_{\max}} \Psi(\theta, \phi, t) X_l^m(\theta)^* e^{-im\phi} \sin \theta \, d\theta \, d\phi$$

Power-spectra of $A_{l,m}(t)$ and $B_{l,m}(t)$ show frequency shift in presence of flows
FLD frequency fit coverage vs. latitude: $B_0$-angle effect