Additive and Multiplicative Systematics
in Ring-Diagram Analysis

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Systematic Error

- Any kind of bias that causes mean of measured values to not match the truth.
- Consider velocities measured with fitting code, compare with known metrics
- Allow dependence on mode, disk position; assume time independent

Systematic Error

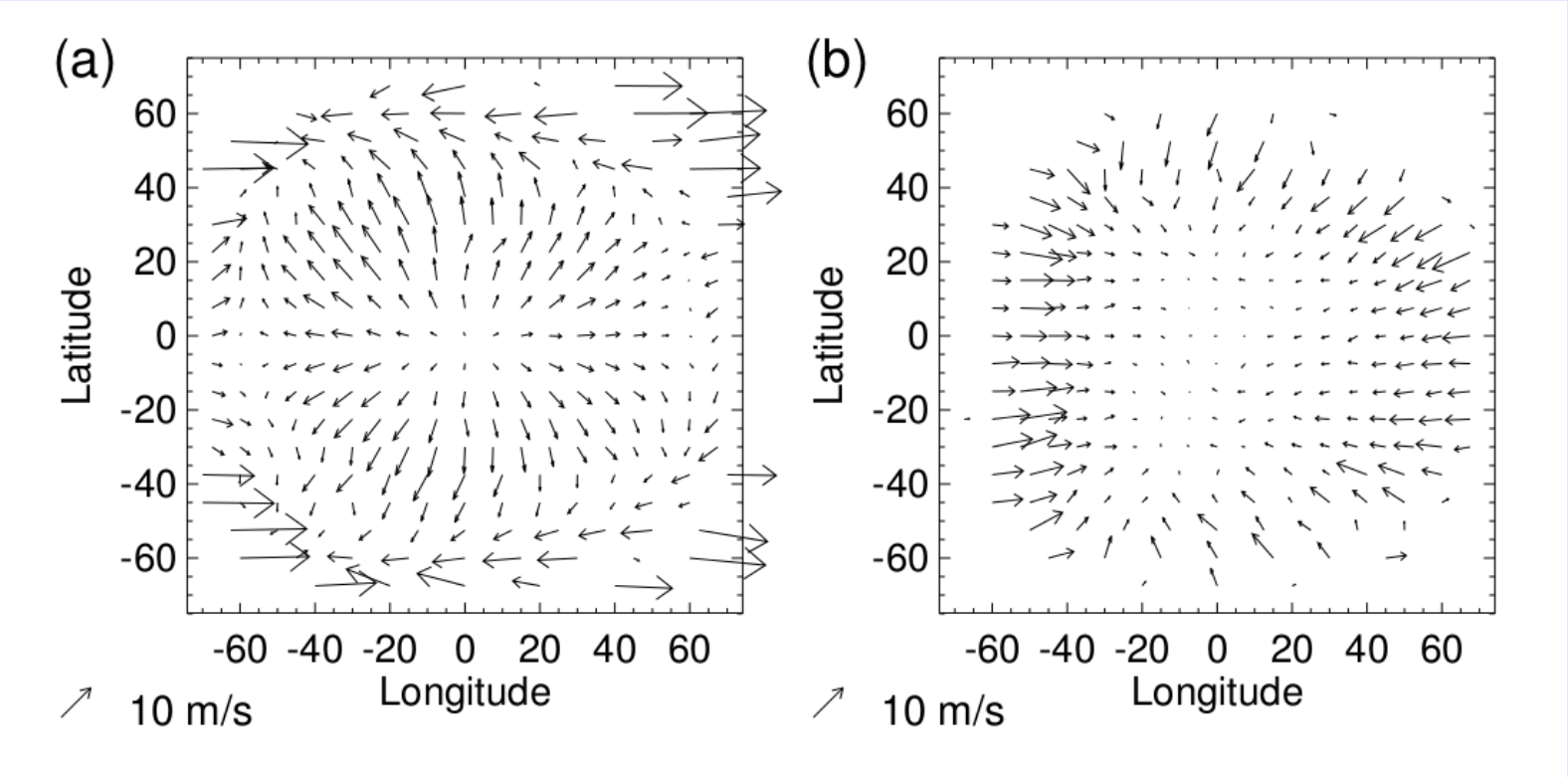
- Assume linear relation between true and measured velocity

$$\tilde{u} = Au + B$$

- Ideal is a = 1, b = 0

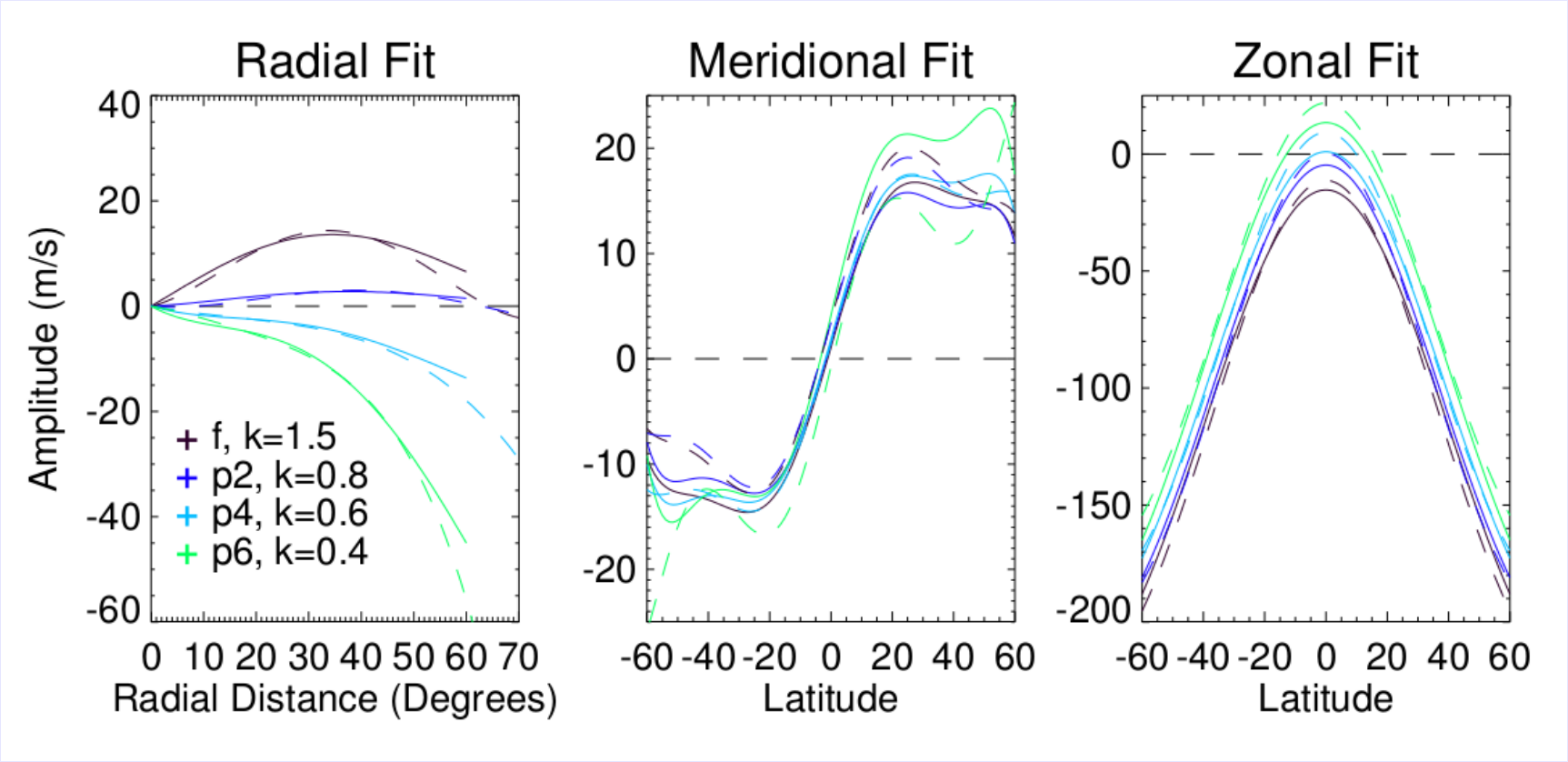
Mapping B Coefs

- See three dominant components:
  - Center-to-limb effect (radial inflow/outflow)
  - Meridional flow (real)
  - Zonal flow (also real)

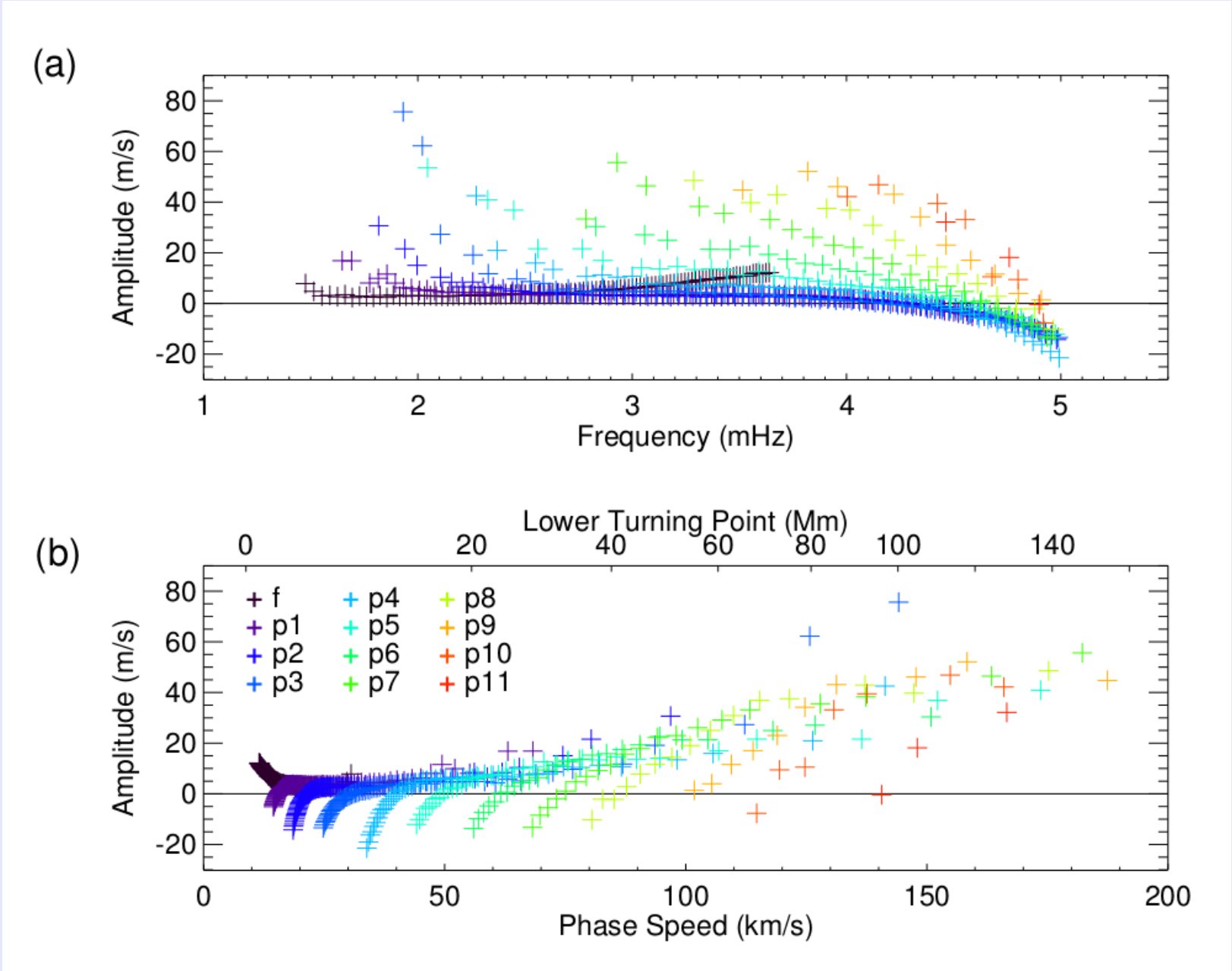


Separating Components

- Use multi-component fit to separate global flows from center-to-limb systematic
- Rely on radial symmetry



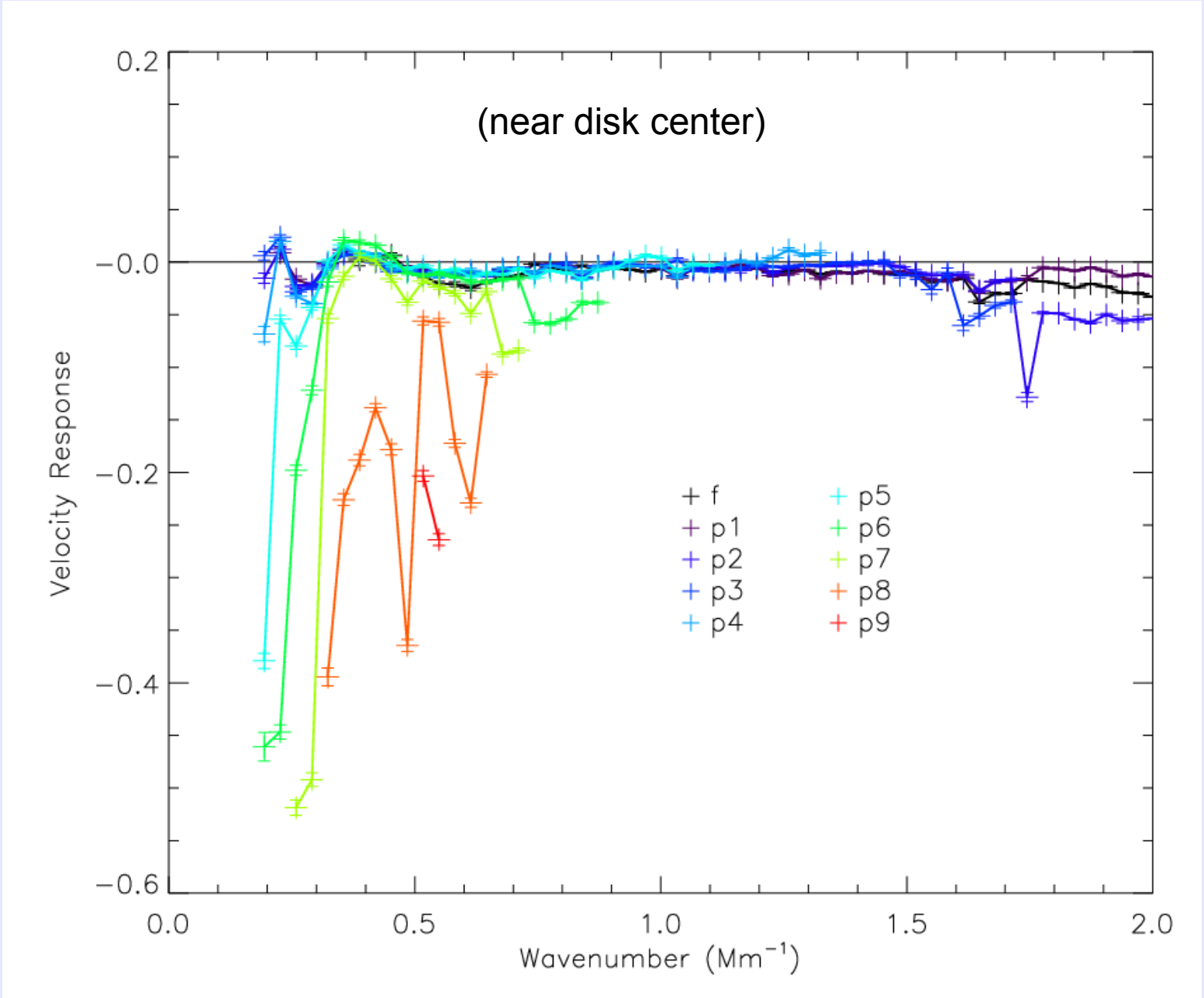
Mapping B Coefs



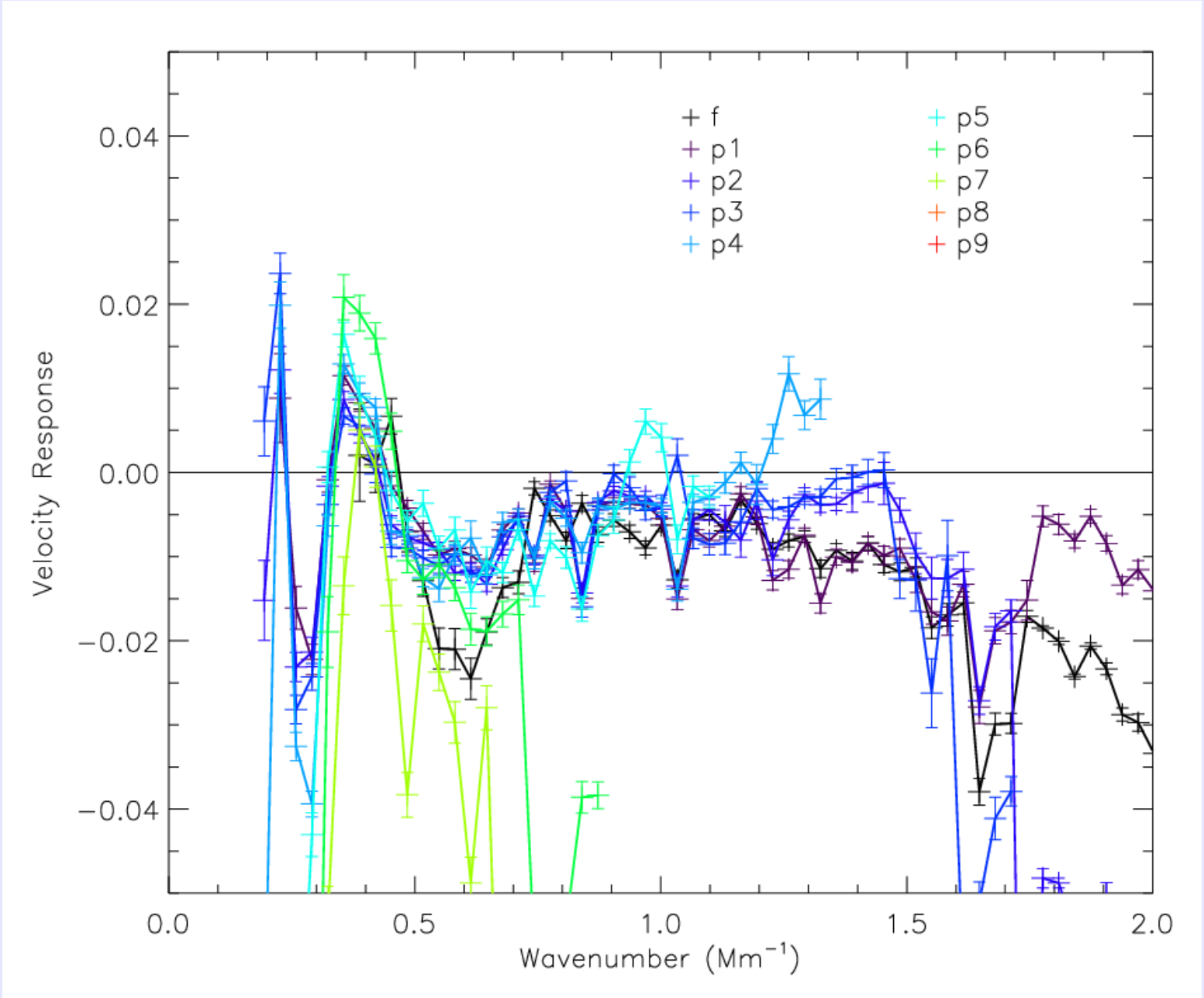
Determining A Coefs

- Don't know the velocity inside the sun accurately enough
- Use differences in measured velocities instead
- Introduce velocities with tracking, fit line to data

Mapping A Coefs



Mapping A Coefs

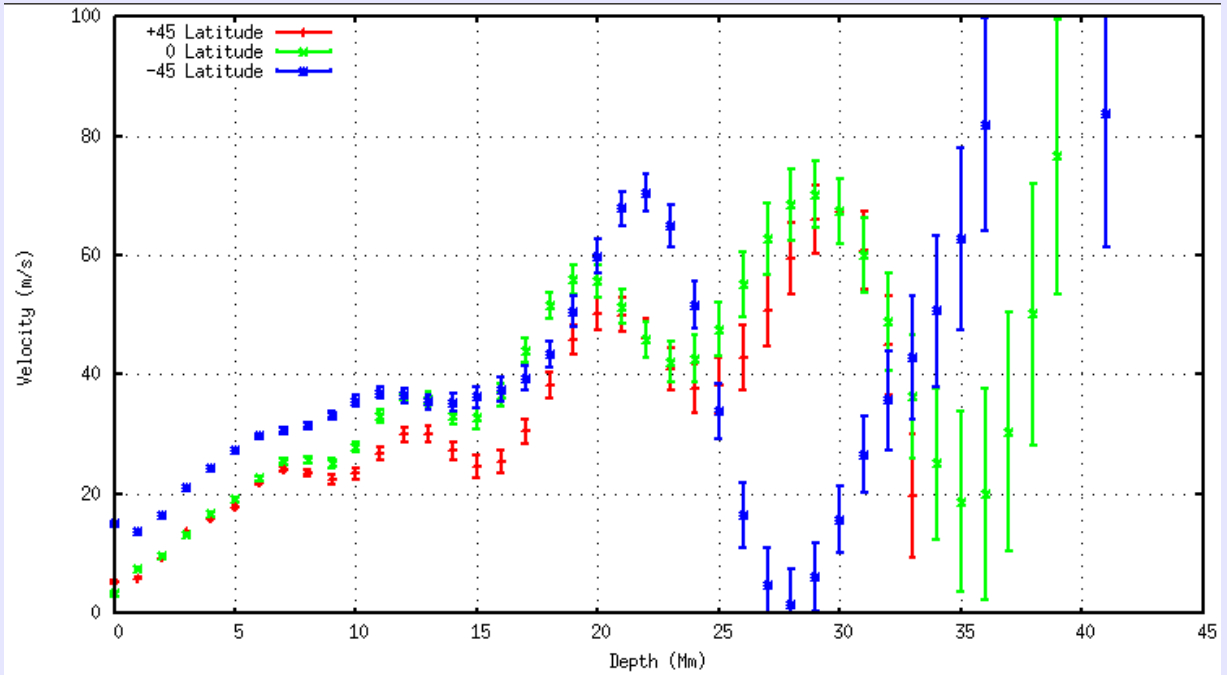


Systematic Amplification

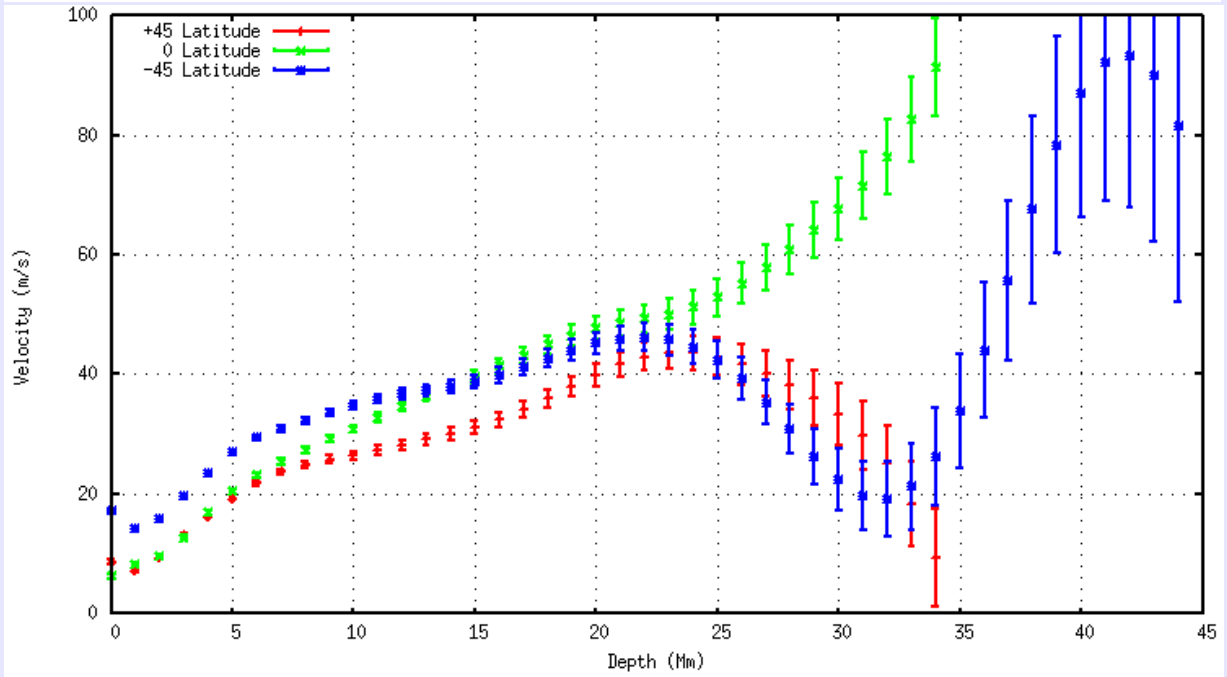
- Inversions will amplify small discrepancies between modes, because that's the point of an inversion.
- Amplification depends on mode-set, regularization

Systematic Amplification

Center-to-limb correction only

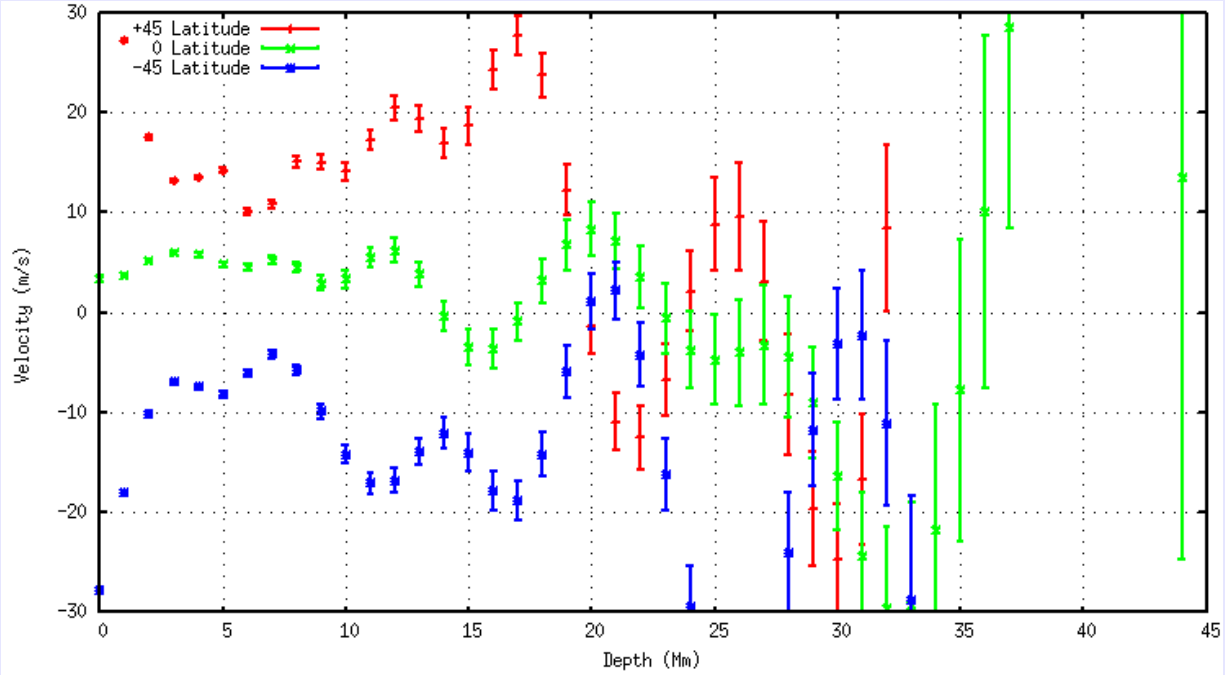


All corrections

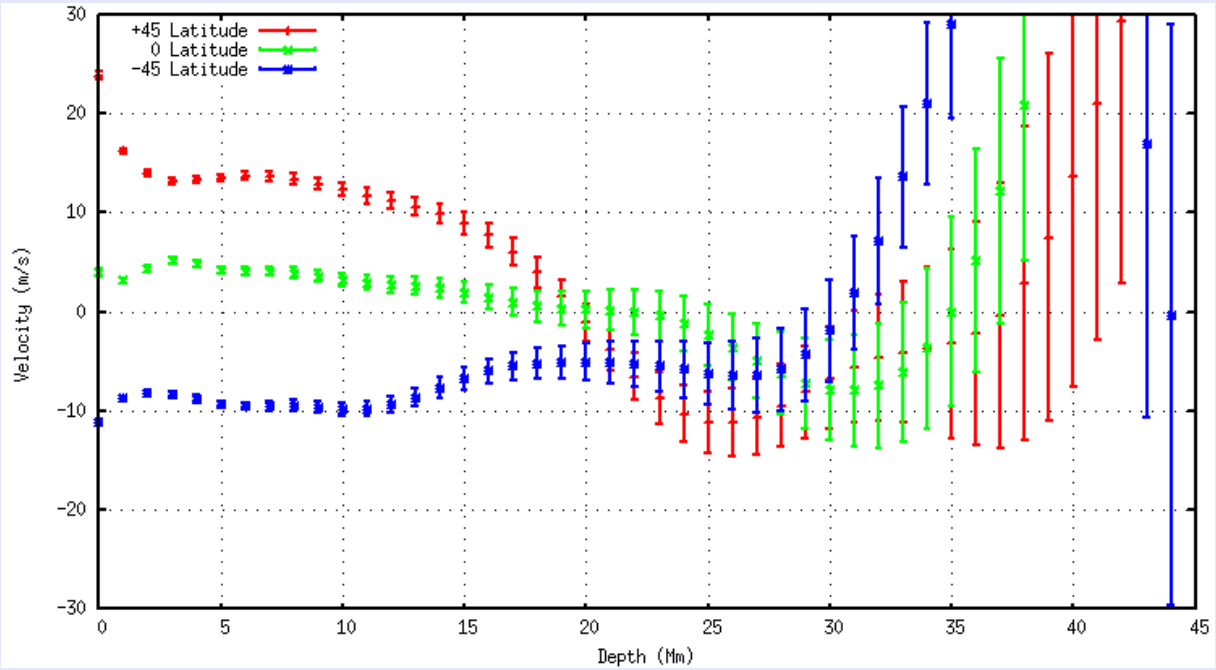


Systematic Amplification

Center-to-limb correction only



All corrections



END

Multi-Component Fit

$$\vec{u}^{(k,n)}(\phi,\lambda) = u_r^{(k,n)}(r)\hat{\boldsymbol{r}} + u_m^{(k,n)}(\phi)\hat{\boldsymbol{\phi}} + u_z^{(k,n)}(\phi)\hat{\boldsymbol{\lambda}},$$

$$u_m^{(k,n)}(\phi) = a_0^{(k,n)} + \sum_{j=1}^{4} a_j^{(k,n)} sin(j\phi),$$

$$u_z^{(k,n)}(\phi) = \sum_{j=0}^{2} b_j^{(k,n)} \phi^{2j},$$

$$u_r^{(k,n)}(r) = \sum_{j=1}^{4} c_j^{(k,n)} r^j.$$